Recent Advances in PDE

Theory, Computations and Applications

8th – 10th June 2017

Book of Abstracts

Department of Mathematics
Indian Institute of Technology Bombay

Day 1: 8^{th} June 2017

Acoustic location of coronary artery disease: computational and experimental aspects

John Whiteman
Brunel University London

Coronary artery disease (CAD) due to the presence of stenoses (partial blockages) of arteries within the heart is a worldwide problem. The development of non- invasive methods for the detection, localisation and characterisation of such stenoses thus remains a major challenge of diagnostic cardiology; see eg. Semmlow and Rahalkar [1]. A possible approach for this is to exploit the effect that, as plaque builds up in a coronary artery, blood flow in diastole past the stenosed region becomes disturbed in the wake and creates abnormal variations in the wall shear stresses. These shears create low amplitude acoustic waves through the thorax which at the chest wall can be measured by placing sensors on the skin. This acoustic surface signature has the potential to provide a cheap non- invasive means of diagnosing CAD.

In this talk we describe an interdisciplinary project which seeks to exploit the above effect. The purpose is to examine the feasibility of identifying wave patterns on the chest surface arising from a wave source within the coronary artery and from these to determine the location and severity of the stenosis (if it is there). For a first stage a cylindrical phantom of tissue mimicking (viscoelastic) material (TMM), agarose gel, is used to represent the chest and with this combined use is made of computational modelling and experimentation of these acoustic effects. Axial symmetry is assumed for the phantom, allowing a 2D section to be used as the problem region. Computational modelling treats the primary (forward) problem, where shear waves arising from a source at a known position in the phantom are produced on the (chest) boundary. Then mathematical inverse problems are solved to locate wave sources in the phantom from the calculated boundary outputs.

For the forward calculation we consider a 2D space-time 'viscodynamic' problem using a wave equation incorporating Zener and Kelvin-Voigt models for viscoelasticity. High order spectral finite element methods in space and high order discontinuous Galerkin finite element discretizations in time, based on normalized Legendre polynomials, are used. Following the technique developed by Werder et al. [2] for the heat equation, we are able in the viscoelastic wave context to decouple the linear systems which arise at each time step. It is well known that dispersion appears in the numerical solution of wave problems using finite element methods and that high order numerical schemes are better suited to control this type of error. The phantom surface results so calculated enable an inverse problem to identify the locations of wave sources using the matlab fminsearch function. Results for both the forward and inverse problems are presented.

In the simultaneous experimental work both axi-symmetric cylindrical specimens of the TMM were used to measure the viscoelastic properties of the material, as well as cuboidal specimens with stenosed tubes embedded along the axis. In the latter fluid flows through the tubes produce acoustic waves that can be detected on the specimen surfaces. For various flow velocities and various levels of stenosis these surface acoustic waves are analysed to indicate how power outputs vary with flow rate, the aim being to identify which frequencies are most relevant to the CAD context and to inform the computational process of what surface signals to apply in the calculations.

Reference:

1. J Semmlow and K Rahalkar, Acoustic detection of coronary artery disease. Annu Rev Biomed Eng 9 (2007), 449-469.
2. T Werder, K Gerdes, D Schotzau and C Schwab, hp-discontinuous Galerkin time stepping for parabolic problems. Comp. Meth. Appl. Mech. Engrg. 190 (2001), 6685 - 6708.

Formulation of sonic boom as a one parameter family of Cauchy problems through Kinematical Conservation Laws

Phoolan Prasad

Indian Institute of Science, Bangalore

I shall discuss an interesting application of 2-D Kinematical Conservation Laws (KCL): a mathematical formulation of a sonic boom. This procedure to calculate signature of a boom is very different from that by engineers.

We first review KCL and its particular case ray equations; and write down the conservation forms of the equations of a weakly nonlinear ray theory (WNLRT) and a shock ray theory (SRT) for a weak shock in a polytropic gas. Then we present a formulation of the problem of sonic boom by a maneuvering aerofoil as a one parameter family of Cauchy problems. The system of equations in conservation form is hyperbolic for a range of values of the parameter and has elliptic nature elsewhere, showing that unlike the leading shock, the trailing shock is always smooth. We also present a few numerical results for the leading shock in the boom and the associated nonlinear wavefront. Numerical procedure is computationally very robust and efficient; and the geometry of the boom is very sharp. Our KCL based method beautifully captures new features, which have been seen in numerical solution of Euler equations, but Euler solutions can not been continued over such long time scales and results are not very sharp. Appearance of kinks (a special feature) is not only very difficult but probably impossible to observed experimentally.

Sonic boom problem has been discussed in great detail by engineers. However, KCL gives a new method, which is more accurate with stable and computationally efficient numerical scheme. I present only the basic theory but it has been extended to the most important case of sonic boom propagation in inhomogeneous media and presented at HYP-2016.

Finite element methods for fourth order elliptic variational inequalities

Susanne Brenner
Louisiana State University

Fourth order elliptic variational inequalities appear in obstacle problems for Kirchhoff plates and optimization problems constrained by second order elliptic partial differential equations. The numerical analysis of these variational inequalities is more challenging than the analysis in the second order case because the complementarity forms of fourth order variational inequalities only exist in a weak sense. In this talk we will present a unified framework for the a priori analysis of finite element methods for fourth order elliptic variational inequalities that are applicable to C^1 finite element methods, classical nonconforming finite element methods, and discontinuous Galerkin methods.

Stabilization of a heat conducting fluid model

Mythily Ramaswamy
TIFR CAM, Bangalore

We consider a fluid model described by Boussinesq system for the velocity, pressure and temperature, in a two dimensional polygonal domain with inflow and outflow boundary conditions. The aim is to stabilize the flow around a steady state by controlling the velocity and temperature in the inflow boundary.

Hodge decomposition methods for electromagnetics

Li-yeng Sung
Louisiana State University

In this talk we present a numerical procedure for solving time harmonic electromagnetic problems that is based on the Hodge decomposition of divergence-free vector fields. In the two dimensional case, it reduces the electromagnetic problems to standard second order elliptic problems that can be solved by many numerical methods. Applications to metamaterials and quad-curl problems will be discussed.

Stabilization of viscoelastic fluids with finite dimensional controllers

Sheetal Dharmatti
IISER, Thiruvananthapuram

In this work, we prove the stabilizability of abstract Parabolic Integro-Differential Equations (PIDE) in a Hilbert space with decay rate by means of a finite dimensional controller in the feedback form. We determine a linear feedback law which is obtained by solving an algebraic

Riccati equation. To prove the existence of the Riccati operator, we consider a linear quadratic optimal control problem with unbounded observation operator. The abstract theory of stabilization developed here is applied to specific problems related to viscoelastic fluids, e.g. Oldroyd B model and Jeffreys model.

An energy-preserving local discontinuous Galerkin method for the Burgers-Poisson equation

Nattapol Ploymaklam
Chiang Mai University

In this talk, I will discuss a local discontinuous Galerkin (LDG) method for solving the Burgers-Poisson equation. This model, proposed by Whitham [Linear and nonlinear waves, 1974] as a simplified model for shallow water waves, admits conservation of both momentum and energy as two invariants. The proposed numerical method is high order accurate and preserves two invariants, hence producing solutions with satisfying long time behavior. The L^2-stability of the scheme for general solutions is a consequence of the energy preserving property. The optimal order of accuracy for polynomial elements of even degree is proven. A series of numerical tests is provided to illustrate both accuracy and capability of the method.

Inhomogeneous Dirichlet boundary condition in the a posteriori error control of the obstacle problem

Gaddam Sharat
Indian Institute of Science, Bangalore

A new and simpler residual based a posteriori error estimator for finite element approximation of the elliptic obstacle problem will be constructed. The results which will be discussed are two fold. Firstly, the influence of the inhomogeneous Dirichlet boundary condition in a posteriori error control of the elliptic obstacle problem will be addressed. Secondly, by rewriting the obstacle problem in an equivalent form, we derive simpler a posteriori error bounds which are free from min/max functions. To accomplish this, we construct a post-processed solution $\tilde{u}_h$ of the discrete solution u_h which satisfies the exact boundary conditions although the discrete solution u_h may not satisfy. We propose two post-processing methods and analyze them. We remark that the results known in the literature are either for the homogeneous Dirichlet boundary condition or that the estimator is only weakly reliable in the case of inhomogeneous Dirichlet boundary condition.

Weak Galerkin finite element methods for elliptic and parabolic interface problems

Papri Roy
Indian Institute of Technology Guwahati

This talk is concerned with numerical approximation of elliptic and parabolic interface problems via weak Galerkin finite element method. Weak Galerkin methods refer to general finite element methods for partial differential equations in which the usual gradient and divergence operators are implemented as distributions in properly defined spaces. Such weak forms give rise to desirable flexibility in enforcing boundary and interface conditions. This method allows the usage of totally discontinuous functions in approximation space and preserves the energy conservation law. A convergence analysis is carried out for elliptic equations with discontinuous coefficients, and optimal order error estimates are established for the corresponding numerical approximation in H^1 norm and L^2 norm. For parabolic interface problems, both continuous and discontinuous time weak Galerkin finite element schemes are developed and analyzed; and optimal order error estimates in both $L^\infty(0,T;H^1)$ and $L^\infty(0,T;L^2)$ norms are established.

Mixed virtual element methods for the Rosenau equation

Sanath Keshav
BITS Pilani, Goa

We consider the Rosenau Equation:

$$u_t + \Delta^2 u_t = \nabla \cdot \vec{f}(u), \quad (x,y) \in \Omega,\ t \in (0,T]$$

with initial condition

$$u(x,y,0) = u_0(x,y), \quad (x,y) \in \Omega$$

and the boundary conditions

$$u = \Delta u = 0 \quad \text{on } \Gamma.$$

where $\Omega = (a,b) \times (c,d)$.

In this talk, we discuss the Mixed Virtual Element Methods for the Rosenau equation, derive the existence uniqueness results and establish *a priori* bounds. The error estimates are derived in both semidiscrete and fully discrete schemes. Finally, we perform some numerical experiments and validate the theoretical results.

Discontinuous Galerkin finite element methods for the two dimensional Rosenau equation

P. Dhanumjaya
BITS Pilani, Goa

The Rosenau equation is an example of a nonlinear partial differential equation, which governs the dynamics of dense discrete systems and models wave propagation in nonlinear dispersive media. Recently, several numerical techniques like conforming finite element methods, mixed finite element methods, orthogonal cubic spline collocation methods, were proposed to find the approximate solution of Rosenau equation. The different conforming finite element techniques which are used to approximate the solution of Rosenau equation needs C^1-interelement continuity condition. The numerical methods based on mixed formulation require C^0-continuity condition.

In this talk, we even relax the continuity condition and discuss the discontinuous Galerkin finite element methods for the Rosenau equation. Theoretical results including consistency, a priori bounds and optimal error estimates are established. Finally, we perform some numerical experiments to validate the theoretical results.

Soliton solutions for modified KdV-Burgers' equation with time dependent coefficients via $\tan(\phi(\xi))$-expansion method

Pallavi Verma
Jaypee institute of information technology

In this talk, modified KdV-Burgers' equation with time dependent coefficients has been investigated to explore some new exact solutions by using $\tan(\phi(\xi))$-expansion method. We have successfully furnished certain new solutions of modified KdV-Burgers' equation with time dependent coefficients in terms of arbitrary functions along with various parameters, which provide further freedom to simulate the desired physical situations. When the parameters are taken as special values, known solitary wave solutions can be derived in a very systematic manner. Moreover, graphical representation of solutions is shown vigorously in order to visualize the behaviour of the obtained solution of the equation. The most accomplishment of this study lies in the fact that this method can be used to obtain exact non-traveling wave solutions of some high-dimensional nonlinear evolution equations.

Pricing European passport option using three time level finite difference scheme

Ankur Kanaujiya
Indian Institute Technology Guwahati

A passport option is a financial derivative with the accumulated gain of a trading account being the underlying security. The partial differential equation (PDE) for the valuation of the European passport option while being linear in the symmetric case (when the risk-free rate is the same as the cost of carry), turns out to be nonlinear for the non-symmetric case (when the risk-free rate is different from the cost of carry). A closed form solution for this valuation problem exists for the symmetric case since the pricing PDE can be reduced to a linear parabolic PDE. However, for non-symmetric case, the pricing PDE is a fully nonlinear degenerate parabolic PDE and is not well-posed in the classical sense. Due to non-smooth payoff, Crank-Nicolson scheme is not suitable for price passport option because it will produce an oscillatory error in the solution as well as larger error in the estimates of Greeks for larger time steps. It is very important to estimate the Greeks correctly because the optimal holding strategy for passport option depends on the Greeks, namely, Delta and Gamma. To resolve these issues we used three-time-level finite difference scheme, which gives the correct estimate of the price as well as Greeks when compared to Crank-Nicolson scheme in the symmetric case. We then extend this and present the numerical price and Greeks for the non-symmetric case which does not have any closed form solution.

Analytic solution of space time fractional advection dispersion equation with retardation for contaminant transport in porous media

Mahaveer Prasad Yadav
MNIT, Jaipur

Advection dispersion equation describes the movement of contaminants along with flowing groundwater at the seepage velocity in porous media. The aim of this work is to find concentration of space time fractional advection dispersion equation with retardation by Caputo fractional derivative and Riesz Feller fractional derivative. The solution of the equation is obtained by applying the Laplace and Fourier transforms in terms of Mittag-Leffler function.

Differential quadrature based numerical schemes for Fisher equation

Ashish Awasthi
NIT Calicut

This work is a comparative study of three numerical schemes, constructed using the combinations of differential quadrature and finite difference methods. Polynomial based differential quadrature method (PDQM) was introduced in the early 1970s by Bellman and his associates. Later, Quan-Chang and Shu modified the procedure for computing the weighting coefficients in PDQM. Many researchers have used PDQM for solving ODEs and PDEs numerically. The interesting fact observed from their studies is the ability of PDQM to produce accurate results, even with less number of grid points. This fact is very useful in problems where initial and boundary conditions are observed discrete data. The Fisher equation is encountered as mathematical model of many real life phenomena like chemical kinetics, population dynamics. In scheme 1, the prototype equation is discretized in the spatial direction using DQM and in the temporal direction using weighted average finite difference method. In scheme 2, the time and space derivatives are approximated with DQM

and central FDM respectively. While in scheme 3, only DQM is used for the discretization of the PDE. For simplicity, the three schemes are labeled as FQTDQS, DQTFDS, and DQTDQS. The results show that the DQTDQS gives much better solutions as compared to the other two. FDTDQS is found to be faster than the other two methods. Moreover, all the three schemes are very simple to program and produce considerably accurate results.

Comparative study of Haar wavelets with Legendre wavelets for fractional order differential equations

Neeraj Kumar Tripathi
IIT(BHU) Varanasi

In this talk, a wavelet operational matrix method based on Haar wavelet and Legendre wavelets method are proposed to solve the fractional order differential equations in the Caputo derivative sense. The methods are used because its computation is simple as it converts the problem into algebraic equations. The known Haar and Legendre Wavelets are presented first then we derived the operational matrix of fractional order derivative. Results obtained when solving some examples are presented to show the efficiency and the accuracy of these two methods.

A uniform Haar wavelet based numerical method for the solution of the Falkner-Skan equation

Harinakshi Karkera
Manipal Institute of Technology

The Haar wavelet method becomes a powerful mathematical tool, when it is successfully coupled with the quasilinearization technique and collocation method. In our work, we study the characteristics of a laminar boundary-layer flow of a viscous and incompressible fluid over a moving wedge theoretically. The Falkner-Skan equation describing this problem is analysed by presenting a simple and accurate solution procedure based on combination of uniform Haar wavelets, quasilinearization and collocation approach. The influence of power law parameter m, variable pressure gradient β and the ratio of free stream velocity to boundary velocity parameter λ are discussed and illustrated through graphs. The residual and error estimates were computed to confirm the validity of the obtained results. Furthermore, a comparison between the present solutions with existing numerical results in the literature is carried out to highlight the benefits and efficiency of proposed method.

Numerical solution of drug release devices by second kind Chebyshev wavelets

Ajay Kumar
IIT(BHU) Varanasi

In this talk, a numerical technique based on shifted second kind Chebyshev wavelets operational matrices of derivatives is used for solving the drug release devices. The mathematical models of the drug release devices involve moving boundary problems. These type of moving boundary problems are highly nonlinear partial differential equations and are not easy to solve. So using a similarity transformation, the nonlinear PDE (drug release devices) is converted into an ordinary differential equation. Numerical results obtained are compared with known analytical solutions.

Boundary condition adapted multiscale representation of derivatives in Daubechies wavelet basis for $L^2[0,1]$ and their applications

Madan Mohan Panja
Visva-Bharati

This work is concerned with the development of a systematic procedure to obtain boundary condition adapted representation of derivatives (differential operators) in the orthonormal wavelet bases of Daubechies family within a bounded domain $\Omega = [0,1]$ in $\mathbb{R}$. Multiscale representation of derivatives

$$y'(x) \equiv (\mathbf{\Phi}_{j_0}, \mathbf{\Psi}_{j_0})\,(\mathbf{y}1_{j_0}, \mathbf{d}1_{j_0})^T\,, \tag{1}$$

$$y''(x) \equiv (\mathbf{\Phi}_{j_0}, \mathbf{\Psi}_{j_0})\,(\mathbf{y}2_{j_0}, \mathbf{d}2_{j_0})^T \tag{2}$$

adopting Dirichlet's boundary conditions

$$y(0) = C^0, \tag{3}$$

$$y(1) = C^1 \tag{4}$$

have been derived first. These representations are then used to reduce the advection-diffusion-transport equation in (1+1)D

$$\bar{R}\frac{\partial c}{\partial t} - \bar{D}\frac{\partial^2 c}{\partial x^2} + \bar{u}\frac{\partial c}{\partial x} + \bar{k}\;c = f(x,t), \quad 0 \leqslant x \leqslant 1,\; t > 0 \tag{5}$$

with the initial-boundary conditions

$$c(0,t) = C^0(t), \quad c(1,t) = C^1(t), \quad c(x,0) = C^I(x) \tag{6a,b,c}$$

to a system of non-homogeneous first order coupled ordinary differential equations (ODEs)

$$\bar{R}\begin{pmatrix} \dot{\overline{\mathbf{y}0}}_{j_0}(t) \\ \\ \dot{\mathbf{d}}0_{j_0}(t) \end{pmatrix} = \mathbb{A}_{j_0}\begin{pmatrix} \overline{\mathbf{y}0}_{j_0}(t) \\ \\ \mathbf{d}0_{j_0}(t) \end{pmatrix} + \mathbf{E}_{j_0}(t). \tag{7}$$

The resulting system of ODEs (7) involving coefficients $\overline{\mathbf{y}0}_{j_0}(t), \mathbf{d}0_{j_0}(t)$ of the multiscale approximation

$$c(x,t) \equiv (\mathbf{\Phi}_{j_0}, \mathbf{\Psi}_{j_0})\,(\mathbf{y}0_{j_0}, \mathbf{d}0_{j_0})^T \tag{8}$$

of solution can be solved globally in time, in general, which helps to prevent the necessary analysis of time discretization such as estimate of step lengths, error etc. In case of convection dominated problems, the system of ODEs (7) can be solved numerically with the aid of any efficient ODE solver. An estimate of L^2-error in wavelet-Galerkin approximation of the unknown solution has been presented. A number of examples e.g., equation having i) separable source term, ii) non-smooth initial datum, iii) non-separable source term including relatively small diffusion coefficient and iv) equation appearing in ground water modelling are considered for numerical illustration. It is found that the scheme is efficient and user friendly. On the generalization of the method towards simultaneous discretization of space-time domain, to advection-diffusion-transport equation with variable coefficients and higher space dimensions, to accommodate Neumann and Robin boundary conditions have been discussed.

A generalized nonconforming finite element method for parabolic problems

Sanjib Kumar Acharya
LNM Institute of Information Technology, Jaipur

In this talk, a class of second order parabolic initial-boundary value problems in the framework of primal hybrid principle is discussed. The interelement continuity requirement for standard finite element method has been alleviated by using primal hybrid method. Finite elements are constructed and used in spatial direction, and backward Euler scheme is used in temporal direction for solving fully discrete scheme. Optimal order estimates for both the semidiscrete and fully discrete method are derived with the help of a modified elliptic projection operator. Numerical results are obtained in order to verify the theoretical analysis.

Convergence analysis of finite element method for a parabolic obstacle problem

Papri Majumder
Indian Institute of Science, Bangalore

A conforming finite element method is proposed and analyzed for numerical approximation of the solution of a parabolic variational inequality of the obstacle problem. The model problem, which is originally proposed using a general obstacle, is reframed as a model problem with zero obstacle but with non-homogeneous Dirichelt boundary conditions. Subsequently the discrete problem is reframed and the error analysis proving the convergence of the method is performed. The analysis requires a positive preserving interpolation with non-homogeneous Dirichlet boundary condition and a post-processed solution that satisfies the boundary conditions sharply. The results in the article extend the results of (Johnson, SINUM, 1976) for a zero obstacle function to a more general obstacle function.

A finite element method for pricing American style options

Lokpati Tripathi
Indian Institute of Technology Bombay

The valuation of American style options is prominent in modern financial theory. The price of an American style option under Black-Scholes model (resp. jump-diffusion model) is governed by parabolic partial differential inequalities (resp. parabolic partial integro-differential inequalities). Since the solutions of such types of inequalities enjoy only limited regularity properties, therefore their numerical approximations become very challenging for financial engineers as well as numerical analysts. The aim of this talk is to present a finite element method combined with appropriate time-stepping procedure for such parabolic partial differential and integro-differential inequalities arising in the pricing of American options. Under realistic regularity assumptions on the data, some error estimates will also be provided. The theoretical findings and the efficiency of the proposed method will be demonstrated by some numerical experiments.

A study on mixed variational like inequalities with convexificator approach and applications

Gayatri Pany
Indian Institute of Technology Bhubaneswar

The aim of this work is to study mixed variational like inequalities using convexificator concept. In general, optimality conditions for variational inequalities are derived using results on optimality conditions for nonsmooth optimization problems in terms of subdifferentials by Clarke, Michel-Penot, Mordukhovich and limiting subdifferential. But sharper results can be obtained by adopting the approach of convexificators, a weaker version to the notion of subdifferentials. Motivated by this aspect, the convexificator approach is exploited in this work. The notion of invexity is extended in the context of convexificators. The mean value theorem for convexificators is extended for invexity. This extended version of mean value theorem is used as the main tool for studying the qualitative and numerical aspects of mixed variational like inequalities. The results obtained are applied to corresponding equilibrium problems.

Existence of the mild solutions for an impulsive fractional differential inclusions in Banach space with sectorial operator

Jitendrakumar G. Panchal
Parul University

Our main contribution in this work is to prove the existence of the mild solutions for an impulsive fractional differential inclusions involving the Caputo derivative using sectorial operator in Banach spaces. The results are obtained by using fractional calculation, operator semigroups and Leray-Schauder's fixed point theorem. An application of our results is also given.

Controllability of neutral functional evolution equations with time varying delays

Falguni S. Acharya
Parul University

In this paper, we study sufficient conditions for controllability of neutral functional evolution equations with time varying delays in Banach space. We shall rely on a Schauder fixed point theorem.

Analysis of optimal control problem for tumor drug delivery mathematical model

Sowndarrajan P. T.
National Institute of Technology Goa

This problem can be illustrated by a mathematical model [1] describing the interaction of cancer cells, normal cells and drug concentration. In this work, we have considered an open bounded domain in an Euclidean space and no smoothness is assumed on the boundary. Further, Neumann boundary condition is assumed on the boundary. Also model incorporates the intrinsic growth rates of cancer cell density, normal cell density and drug concentration. The death rate of the tumor cells due to competition for resources with the normal tissue, the death rate of tumor cells due to drug treatment and the death of normal tissues due to competition with tumor cells is also included in this model. The rate at which the drug is being injected, a function of time and state variables and it is the control variable for an optimal control system. We prove the existence of solutions for the direct and the adjoint problem using the Faedo-Galerkin method and also verified the necessary first-order optimality condition satisfied for the given control problem.

Reference:

1. S.P. Chakrabarty and F.B. Hanson, Distributed parameters deterministic model for treatment of brain tumors using Galerkin finite element method, Mathematical Biosciences, 219(2009), 129 - 141.

A mixed and discontinuous finite volume approximation of optimal control problem governed by two-phase incompressible immiscible flow in porous media

Ruchi Sandilya
IIST, Thiruvananthapuram

In this talk, we introduce a combined method consisting of the mixed finite volume element and discontinuous Galerkin finite volume element methods in space, and backward Euler method in time for solving the optimal control problem governed by a coupled system of immiscible displacement equations. The resulting discrete system is non-symmetric and we have employed optimize-then-discretize approach to approximate the control problem. The estimates of error between a given local reference solution of infinite dimensional optimal control problem and their approximations in suitable norms are derived with the optimal orders and minimum regularity requirements. Numerical experiments are presented to illustrate some of the theoretical results.

Optimal control of the velocity term in plate equation with multiplicative control

Anil Kumar Pundir
BITS Pilani KK Birla Goa Campus

We consider an optimal control problem for a plate equation with multiplicative distributive control. Multiplicative control is the coefficient $u(x,t)$ of velocity term w_t. Using multiplicative control, we bring the state solution to the desired profile under a quadratic cost of control. We prove the existence of a optimal control. Criterion for characterization of the optimal control is provided.

Day 2: 9th June 2017

From Poincaré to Saint-Venant: via Donati, Lions and Körn

S. Kesavan
Indian Institute of Technology Madras

A proof of the 'H^{-1} version' of the Poincaré lemma, valid in all space dimensions, will be presented. It will also be shown that the Saint-Venant conditions for a symmetric matrix field to be the strain tensor of a displacement field is the 'matrix analogue' of the Poincaré lemma and the 'H^{-1} version' will be proved along the same lines. The space of strain tensors will also be characterized via orthogonality relations, thus generalizing results of Donati (smooth case) and Ting (L^2 case).

The arguments used will bring out the intimate relationship between several fundamental results like J. L. Lions' lemma, Körn's inequality.

Adaptive least-squares finite element methods

Carsten Carstensen
Humboldt University of Berlin

The least-squares functional is a reliable and efficient error estimator with global upper and lower bounds and can be very accurate. The elementwise contributions to the global L^2 norm serves well as refinement indicators in adaptive mesh-refining algorithms but the convergence analysis is less well understood. Those local contributions do not involve an explicit mesh-size factor and hence

their reduction is unclear. The paper Bringmann, Carstensen, and Park (Numer. Math. 2017) guarantees the plain convergence for a bulk parameter close to one and that is far away from the arguments for rate optimality.

The axioms of adaptivity in Carstensen, Feischl, Page, and Praetorius (Comp. Math. Appl. 2014) are not available and an alternative error estimator is derived and exploited in Carstensen and Park (SIAM J. Numer. Anal. 2015) and enforces a separate marking strategy with an overall abstract theory by Carstensen and Rabus (arXiv 1606.02165 2016).

The presentation discusses on all those aspects for the Laplace, the Stokes and the Lamé-Navier equations as in Bringmann and Carstensen (Numer. Math. 2017). Numerical experiments confirm the proven optimal convergence rates. If time permits, a nonlinear model problem shall be discussed as well.

Local stabilization of 2D and 3D fluid-structure models

Jean-Pierre Raymond
Université Paul Sabatier Toulouse

We shall address the problem of stabilizing systems coupling the incompressible Navier-Stokes equations in a 3D domain with either an elastic structure immersed in the fluid or a structure located at the boundary of the fluid domain. The goal is to find a control acting either at the boundary of the fluid domain, or in the structure equation, able to stabilize the underlying coupled system, with an arbitrarily prescribed exponential decay rate, around an unstable stationary solution of the coupled system. I shall present some extensions of recent results obtained for 2D (in collaboration with M. Ndiaye, M. Fournié,) or 3D domains (in collaboration with D. Maity, A. Roy, and separately with M. Vanninathan), and some numerical simulations in 2D showing the efficiency of feedback control laws to stabilize coupled systems.

a posteriori error analysis of linear parabolic interface problems: a reconstruction approach

Rajen Sinha
Indian Institute Technology Guwahati

We derive residual-based a posteriori error estimates of finite element method for linear parabolic interface problems in a two-dimensional convex polygonal domain. Both spatially discrete and fully discrete approximations are analyzed. While the space discretization uses finite element spaces that are allowed to change in time, the time discretization is based on the backward Euler approximation. The salient features of the a posteriori error analysis includes: (i) an appropriate adaptation of the elliptic reconstruction technique, and (ii) new approximation results for Scott and Zhang interpolation operator. We use only an energy argument to establish a posteriori error estimates with optimal order convergence in the $L^2(H^1)$-norm and almost optimal order in the

$L^\infty(L^2)$-norm. The interfaces are assumed to be of arbitrary shape but are smooth for our purpose. Numerical results are presented to validate our derived estimators.

Dispersive Approximation in Finely Periodic Media

M. Vanninathan
Indian Institute Technology Bombay

Deriving approximate models for acoustic waves in heterogeneous media is a classical problem. A first approximation is provided by the homogenized model which captures an overall modification of the wave speed of propagating waves caused by heterogeneity. However, as shown by experiments, small scales undergo dispersion and so the above model is not adequate. Finding satisfactory dispersive approximation for general heterogeneous media remains an open problem. For periodic media, however, there is a possibility of applying Bloch waves to find such an approximation. This talk is about describing this application, highlighting difficulties and solutions.

Finite element computations using locally supported biorthogonal systems

Bishnu Lamichhane
University of Newcastle

I focus on the applications of locally supported biorthogonal basis functions for various problems in computational mechanics. In particular, the biorthogonal basis functions will be applied to Stokes, biharmonic and elasticity equations, to resolve the non-conforming triangulations and to recover the gradient. In the second part of the talk, I will briefly discuss the construction of these locally supported biorthogonal basis functions.

A well-balanced and asymptotic preserving IMEX Runge -Kutta scheme for the Saint-Venant system

K. R. Arun
IISER, Thiruvananthapuram

We derive an implicit-explicit (IMEX) finite volume scheme for the Saint-Venant system of shallow water equations. We will, in particular, concentrate on a singular limit of low speed flows when the Froude number goes to zero. In order to efficiently resolve the slow dynamics, we split the whole nonlinear system into a stiff linear part governing the gravitational waves and a non-stiff nonlinear part that models nonlinear advection effects. We use a stiffly accurate second order IMEX Runge-Kutta scheme for time discretisation, to approximate stiff linear operator implicitly and the non-stiff nonlinear operator explicitly. It will be shown that the resulting scheme is well balanced for the lake at rest equilibrium and asymptotic preserving for the zero Froude number limit.

Virtual element method for Benjamin-Bona-Mahony (BBM) equation

Balaje K.
BITS Pilani, Goa Campus

We consider the BBM Equation:

$$u_t - \Delta u_t = \nabla \cdot \vec{f}(u), \quad (x,y) \in \Omega,\ t \in (0,T]$$

with initial condition

$$u(x,y,0) = u_0(x,y), \quad (x,y) \in \Omega$$

and the boundary conditions

$$u = 0 \quad \text{on } \partial\Omega,$$

where $\Omega = (a,b) \times (c,d)$.

In this talk, we discuss the Virtual Element Methods for the BBM equation and derive the existence uniqueness results and establish *a priori* bounds. The error estimates are derived in both semidiscrete and fully discrete schemes. Finally, we perform some numerical experiments and validate the theoretical results.

Discontinuous Galerkin methods for Keller-Segel chemotaxis system with chemotaxis sensitivity and cross-diffusion

Gurusamy A.
Bharathiar University

In this work, both symmetric and non-symmetric interior penalty discontinuous *hp*-Galerkin methods are applied to Keller-Segel chemotaxis system with cross-diffusion. The spatial discretization of the system is based on the discontinuous Galerkin method and the temporal discretization is based on the explicit fourth order Runge-Kutta scheme. We prove the existence of solutions to the discrete problem using Schauder's fixed point theorem. *a priori* error estimates are derived. Numerical experiments illustrating theoretical results are provided.

An expanded mixed FEM for a nonlinear parabolic problem

Nisha Sharma
M.C.M. D.A.V. College for Women, Chandigarh

In this work, an expanded mixed finite element method with lowest order Raviart-Thomas elements is developed and analyzed for a class of nonlinear and nonlocal parabolic problems. After obtaining some regularity results for the exact solution, a priori error estimates for the semidiscrete problem are established. Based on a linearized backward Euler method, a complete discrete scheme is proposed and a variant of Brouwer's fixed point theorem is used to show the existence of a fully discrete solution. Further, a priori error estimates for the fully discrete scheme are established. Finally, numerical experiments are conducted to confirm our theoretical findings.

Mortar finite element methods for hyperbolic problems

Ajit Patel
LNM Institute of Information Technology, Jaipur

Non-conforming domain decomposition method known as mortar method is useful where the domain of PDEs consists of heterogeneous materials. The mortar finite element method is applied for spatial discretization and a finite difference scheme is used for time discretization of a class of hyperbolic problems. Optimal error estimates in $L^\infty(L^2)$ and $L^\infty(H^1)$ norms for both semidiscrete and fully discrete schemes are discussed. The results of numerical experiments support the theoretical results obtained.

Quarter of circular plate with exponential thickness variation

Neetu Singh
Babasaheb Bhimrao Ambedkar University, Lucknow

The transverse vibration of a circular plate plays very important role in the design of naval structure, aircraft design, etc. First few frequencies play crucial role for getting best structural design. The present research work considers quarter of a circular plate with thickness having exponential variation. The Rayleigh-Ritz method is used for finding the numerical solutions and eigenvalues. The solution in the form of eigenvalues is further computed by generalized Jacobi method which gives first few frequencies. Final results are computed for clamped, simply-supported, and completely-free plates. In special case, results are computed for uniform thickness and represented in the form of tables and graphs.

Orthogonal spline collocation methods for the two dimensional parabolic interface problem

Santosh Kumar Bhal
BITS Pilani K K Birla Goa Campus

The two-dimensional parabolic interface problem appears in many physical and engineering applications, such as the continuous casting in the metallurgical industry, the freezing process of perishable foodstuffs in food engineering, and the magnetic fluid hyperthermia treatment of cancer. The analysis of conductive heat transfer process over composite media is indispensable in these applications.

In this talk, we present Orthogonal Spline Collocation Methods (OSCM) for solving two dimensional parabolic interface problem. In particular, two dimensional parabolic interface equation is discretized using OSCM with monomial cubic basis function in space coupled with Crank-Nicolson method in time. We have studied existence and uniqueness of solution to the problem, and numerical errors and convergence rates are tested in an example.

Invariant solutions of Boiti-Leon-Manna-Pempinelli equation

Atul Kumar Tiwari
MNNIT Allahabad

Invariant solutions of the (2 + 1)-dimensional Boiti-Leon-Manna-Pempinelli equation are obtained by using the similarity transformation method via Lie group theory. The Boiti-Leon-Manna-Pempinelli equation has been reduced to a new partial differential equation with less number of independent variables after applying similarity transformations method successfully. Again using the similarity transformation method the new partial differential equation is reduced to an ordinary differential equation. The ordinary differential equation provides invariant solutions in terms of arbitrary functions of y and t. Elastic behaviour of multisolitons and parabolic nature of solutions are analyzed physically through their evolutional profiles.

Solution of multilinear systems using the Moore-Penrose inverse of tensors

Ratikanta Behera
IISER Kolkata

In recent years the concept of tensor-structured numerical methods has opened new perspectives for solving the basic equations of mathematical physics in R^d, $d \geqslant 3$. In this talk, we focus on solution of multilinear systems using the Moore-Penrose inverse of tensors with the Einstein product. Here, we elaborate a few characterizations of different generalized inverses of tensors to solve multilinear systems, where Sylvester equation plays an important role. Further, generalized inverses are also applied to the generalized least squares problem and several useful identities for solution of multilinear systems are proved.

A second order semi-implicit scheme for the linear wave equation system, accurate at the low Mach number limit

Saurav Samantaray
IISER, Thiruvananthapuram

Low-Mach number limit is a singular limit for the purely hyperbolic compressible equations, which changes their type to mixed hyperbolic-elliptic equations. Due to this asymptotic convergence, numerical schemes designed only for compressible equations suffer from several pathologies: namely, reduction of order, lack of stability and inconsistency. In the present work we consider a framework for analysing the above three difficulties by taking the wave equation with advection as a prototype model. Guided by a systematic multi-scale asymptotic expansion, we split the fluxes into the so-called stiff and non-stiff parts. The non-stiff part is so designed that its Jacobian matrix always has finite eigenvalues, facilitating the use of standard upwind discretizing procedures. On the other hand, the stiff part constitutes a wave equation system with very large wave speeds. We use second order implicit-explicit (IMEX) Runge-Kutta (RK) scheme for discretization, therein the non-stiff part is treated explicitly and the stiff part implicitly. To simplify the implicit nature of the scheme and the solution of the resulting algebraic equations, only diagonally implicit RK schemes are studied. We present the results of numerical case studies which confirm the efficacy of our flux splittings and accuracy and stability of the resulting IMEX RK schemes.

High order two-level implicit difference formulas for 2D fourth-order parabolic partial differential equations

Deepti Kaur
University of Delhi

The current work proposes two-level implicit difference formulas for the solution of fourth-order parabolic partial differential equations. The proposed methods are fourth order accurate in space and second order accurate in time involving nine spatial grid points of a compact stencil. Also, we develop the alternating direction implicit scheme for a fourth-order linear parabolic equation which is shown to be unconditionally stable. Some numerical examples are included to demonstrate the validity and applicability of the method.

Numerov type discretization for the solution of one-dimensional hyperbolic equations

Rajni Arora
University of Delhi

In this work, we propose a new three-level implicit Numerov type discretization for the solution of one-dimensional hyperbolic telegraphic equation subject to appropriate initial and Dirichlet boundary conditions. The proposed method is shown to be unconditionally stable by matrix method. Numerical examples are given to demonstrate the efficiency and accuracy of the proposed method.

A high-order compact discretization for three-dimensional convection-diffusion problems on a quasi-variable mesh network

Navnit Jha
South Asian University

We describe a new high-order accurate compact finite-difference discretization for the numerical solution of three-dimensional convection-diffusion problems. The compact operators are defined on a quasi-variable mesh-network with same order and accuracy, as obtained by the help of central difference and averaging operators on a uniform meshes. Subsequently, a fourth-order difference scheme is developed; to get a numerical accuracy of order four on quasi-variable meshes as well as on uniform meshes. The convergence analysis of fourth-order compact scheme is described in detail by the help of matrix analysis. Some examples are provided related with convection-diffusion equations to present performance and robustness of the proposed scheme.

Mathematical model to analyze the dynamic response due to load moving in irregular initially stressed heterogenous granite rock medium

Anil Negi

Indian Institute of Technology (Indian School of Mines) Dhanbad

In the present work, the effect of moving load in an irregular initially stressed granite rock medium with rough surface has been investigated by examining induced incremental normal and shear stresses. The closed form expressions of normal and shear stresses induced due to moving load have been derived by means of analytical treatment for the problem. The effect of various affecting parameters viz. irregularity depth, heterogeneity, irregularity factor, frictional coefficient and horizontal initial stresses (compressive and tensile) on induced normal and shear stresses have been examined. Using numerical computation, the obtained results have been demonstrated graphically.

Non-uniform grid compact finite difference scheme for mildly non-linear three dimensional singular elliptic equations

Bhagat Singh

South Asian University

We describe a new fourth-order accurate compact finite differencing scheme based on a non-uniform grid for obtaining numerical solution of three dimensional singular elliptic equations. The difference schemes yield an accuracy of order four on non-uniform meshes as well as on uniform meshes. The peculiar behaviour of quasi-variable mesh parameters in all three directions helps in resolving layer behaviour of the problems. The method is analyzed on convection-diffusion problems and other physical problems. The accuracy in solution values and computational order of convergence have been presented to show the applicability and superiority of the proposed scheme.

Fractional order generalized thermoelastic response in a half space due to a periodically varying heat source

Jyoti Verma

Nagpur University

The present work is concerned with the solution of a fractional order thermoelastic problem of a two dimensional infinite half space under axisymmetric distributions in which lower surface is traction free and subjected to a periodically varying heat source. The thermoelastic displacement, stresses and temperature are determined within the context of fractional order thermoelastic theory.

To observe the variations of displacement, temperature, and stress inside the half space we compute the numerical values of the field variables for copper material by utilizing Gaver-Stehfast algorithm for numerical inversion of Laplace transform. The effects of fractional order parameter on the variations of field variables inside the medium are analyzed graphically.

Double diffusive mixed convection flow from a vertical exponentially stretching surface in presence of the viscous dissipation

Latha D. N.
Karnatak University, Dharwad

This work is devoted to obtain non-similar solutions for the effect of viscous dissipation on the steady double diffusive mixed convection flow over a vertical exponentially permeable stretching surface. The non-linear partial differential equations governing the flow, thermal and species concentration fields are written in the non-dimensional form by using suitable group of transformations. The final non-dimensional set of coupled partial differential equations is solved using the implicit finite difference method in combination with the Newtons linearization technique. The effects of various non-dimensional physical parameters on velocity, temperature and species concentration fields are discussed. The presence of the suction/injection at the surface expedites the mass transfer phenomena. The numerical results in terms of the skinfriction coefficient, the rate of heat transfer in terms of local Nusselt number, and mass transfer rate in terms of Sherwood number are shown graphically for various physical parameters involved in the problem. The present results are compared with previously published work and these comparisons are found to be in excellent agreement.

Normal wave interaction with finite floating rigid dock in presence of trapezoidal trench

Amandeep Kaur
Indian Institute of Technology Ropar

Over the years, several researchers have studied the interaction of water waves with floating rigid dock used as a breakwater in the water of uniform depth. However, in nature, the bottom bed is often found to be undulating type. One such undulation shape is trapezoidal trench.

The two-dimensional problem of normal water wave scattering by a floating dock of finite width in presence of a trapezoidal trench is studied. The problem is handled for its solution using the step method. Here the sloped wall of trapezoidal trench is approximated using successive steps. When the bottom has an undulation of arbitrary shape, the shape can also be approximated with successive steps.

Employing the matching eigenfunction expansion, boundary value problem is reduced to a system of linear algebraic equations. The system of equations is solved to find out the physical

quantities, namely, the reflection and transmission coefficients associated with reflected wave and transmission wave respectively. In addition, the force and moment on the floating dock are evaluated for various values of the system parameters. The results are compared for both symmetric and asymmetric trapezoidal trenches. An important relation in the study of water waves namely the energy balance relation is also derived and used to check the accuracy of the computational results.

Effect of viscosity and anisotropy on propagation of Rayleigh type wave

Snehamoy Pramanik
Indian Institute of Technology (Indian School of Mines), Dhanbad

The present work is incorporated with a mathematical model of Rayleigh wave propagation in a viscoelastic earth crust lying over an anisotropic elastic half space. The displacement components of viscoelastic and anisotropic media is deduced separately. The phase velocity has been calculated numerically subject to appropriate boundary conditions. The effect of various parameters on phase velocity as well as dispersion equation are illustrated through graphs using "Mathematica".

Day 3: 10^{th} June 2017

Applications of Hamilton-Jacobi equations in shape from shading

G.D. Veerappa Gowda
TIFR, Bangalore

Hamilton-Jacobi equations have wide applications in numerous fields of science such as classical mechanics and geometrical optics in physics. In this talk we emphasize upon both theoretical and numerical perspectives for this first order non-linear partial differential equations, especially focussing on the application in the shape from shading *i.e.*, to recover the shape of 3-dimensional object from 2-dimensional informations.

Quasi-optimality in parabolic spatial semi-discretizations

Andreas Veeser
University of Milan

We analyze the interplay of the time derivative and the spatial discretization of parabolic initial-boundary value problems. Addressing time-independent and time-dependent spatial discretizations, our focus is on quasi-optimality and best error localization. Best error localization means that the best error over the whole domain is equivalent to an l_2-norm of best errors over small subdomains, which ideally are mesh elements. The key tool for quasi-optimality is the inf-sup theory, with a following formula for the quasi-optimality constant which is of independent interest.

On the wave equations of Kirchhoff-Narasimha and Carrier

A. S. Vasudeva Murthy
TIFR Bangalore

A nonlinear nonlocal wave equation modelling the coupling between transverse and longitudinal vibrations was derived by Carrier in 1945. In 1968 Narasimha derived a similar equation but with a different nonlinearity (nowadays referred as Kirchhoff type nonlinearity). These two equations are solved numerically and compared.

Local best approximation by finite element spaces

Thirupathi Gudi
Indian Institute of Science Bangalore

In the spirit of [Veeser, *Found. Comp. Math.*, 16: 723–750, 2016], we prove that the global best finite element approximation is controlled by the sum of local best approximations without continuity requirements that are imposed in the global finite element space. This result, which is proved in a general framework under a structural hypothesis to enable applications to a variety of finite element spaces, is accomplished by using interpolation operators, averaging operators, trace and inverse inequalities and Poincaré type inequalities. Application of the general framework to some of the finite element spaces are discussed by verifying the structural hypothesis in each of those contexts. The results may be useful in proving the quasi-optimality of adaptive finite element methods.

Numerics of nonlinear biharmonic problems of first kind: application to Navier-Stokes equations of motion

R. K. Mohanty
South Asian University

In this talk, we will present compact second and fourth order accurate numerical methods for the solution of nonlinear biharmonic equations subject to Dirichlet and Neumann boundary conditions. The proposed method will be then employed to solve Navier-Stokes equations of motion in terms of streamfunction-velocity formulation, and the lid-driven square cavity problem. We will discuss numerical methods for the estimates of vorticity function. Computational results will be presented to demonstrate the applicability of the proposed methods.

Parameter uniform numerical schemes for singularly perturbed differential difference equations

Kapil Sharma
South Asian University

Singularly perturbed differential difference equations provide a significant tool to simulate many real life phenomena. The singularly perturbed differential equation is characterized by a parameter, which is multiplied in a highest order derivative term. The solution of this class of differential equations exhibits layer behavior in narrow regions. These narrow regions are known as layer region and rest part of the domain is known as outer region.

If the differential and difference terms occur simultaneously in an equation, this class of differential equations is classified as differential difference equations. If the highest order derivative term in differential difference equations is multiplied by a small parameter and the solution of the problem exhibits abnormal behavior in narrow regions, then this class of differential equations is named as singularly perturbed differential difference equations. Till now there is no method available to find their exact solution. Therefore, we have to rely on the approximate methods for such type of problems.

To develop the approximate methods for this class of differential equations, the researchers encounter with two major difficulties, namely, due to presence of singular perturbation parameter and the presence of differential and difference terms simultaneously.

In this talk, there is an attempt to introduce the audience to this class of differential equations. Further, some recent work on the development of parameter uniform numerical methods to find approximate solutions of boundary value problems for singularly perturbed differential difference equations is presented.

Alternating direction implicit finite element Galerkin methods for cubic Schrödinger equation

Morrakot Khebchareon
Chiang Mai University

We formulate and analyze the fully discrete approximate solution of the cubic Schrödinger equation in two space variables on the unit square written as a Schrödinger-type system. The finite element Galerkin method is used for the spatial discretization, and the time-stepping is done with an alternating direction implicit Crank-Nicolson method. We demonstrate the existence and uniqueness of the approximations, and prove that the schemes are stable and are of optimal accuracy in time and in the H^j, $j = 0, 1$ norms in space. Numerical results are presented which support the theory.

Finite element approximation to global stabilization of BBM-B type equation by nonlinear boundary feedback control

Sudeep Kundu
Indian Institute of Technology Bombay

In this talk, we discuss global stabilization results for the Benjamin-Bona-Mahony-Burgers (BBM-B) type equation using nonlinear Neumann boundary feedback control law. Then, based on C^0-conforming finite element method, global stabilization results for the semidiscrete solution are analyzed. Further, introducing an auxiliary projection, optimal error estimates in $L^\infty(L^2)$, $L^\infty(H^1)$, and $L^\infty(L^\infty)$-norms for state variable are obtained. Further, superconvergence results are derived for the feedback control laws, which preserve exponential stabilization property. All the results are even valid for $\kappa \to 0$. Finally, some numerical results are discussed to confirm our theoretical findings.

Backward Euler schemes for the Kelvin-Voigt viscoelastic fluid flow model

Ambit K. Pany
SOA University, Bhubaneswar

Let Ω be a bounded domain in $\mathbb{R}^d$ ($d = 2$ or 3) with boundary $\partial\Omega$. Consider the following system of equations described by the Kelvin-Voigt viscoelastic fluid flow model: Find a pair $\mathbf{u}, p$ such that

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} - \kappa \Delta \mathbf{u}_t - \nu \Delta \mathbf{u} + \nabla p = \mathbf{f}(x,t), \;\; x \in \Omega, \;\; t > 0, \tag{0.1}$$

with incompressibility condition

$$\nabla \cdot \mathbf{u} = 0, \;\; \mathbf{x} \in \Omega, \, t > 0, \tag{0.2}$$

and initial and boundary conditions

$$\mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0 \quad \text{in } \Omega, \qquad \mathbf{u} = 0 \quad \text{on } \partial\Omega, \;\; t \geqslant 0. \tag{0.3}$$

Here, $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$ denotes the velocity vector, $p = p(\mathbf{x}, t)$ is the pressure, $\nu > 0$ represents the kinematic coefficient of viscosity and κ is the retardation in time parameter. we discuss backward Euler method for the equations of motion arising in the Kelvin-Voigt viscoelastic fluid flow model with non-zero forcing function $\mathbf{f} \in \mathbf{L}^\infty(\mathbf{L}^2)$. After deriving *a priori* bounds for the fully discrete schemes which are valid uniformly in time, the existence of a discrete global attractor is established. Then, optimal *a priori* error estimates are obtained, whose bounds may depend exponentially in time. Under uniqueness condition, these estimates are shown to be uniform in time. Finally numerical experiments are conducted which confirm our theoretical findings.

Domain decomposition and mixed finite element methods for elliptic problems

Debasish Pradhan
Defence Institute of Advanced Technology, Pune

This work deals with the analysis of an iterative non-overlapping domain decomposition method for elliptic problems based on a mixed finite element method. We have used Robin-type boundary conditions to obtain the transmission data on the inter-subdomain boundaries. In order to derive the corresponding discrete problem, we have used the Raviart-Thomas spaces of lowest order RT_0. The convergence of the iterative scheme is obtained by proving that the spectral radius of the matrix associated with the fixed point iterations is less than 1. The rate of convergence is derived which is of order 1-$O(h^{1/2}H_*)$, where h is the finite element mesh parameter and H_* is the minimum diameter of the subdomains. The numerical experiments confirm the theoretical results established in this work.

Patchwise local projection stabilization for convection-diffusion problem

Asha Kisan Dond
Indian Institute of Science, Bangalore

The standard Galerkin finite element methods fail to provide a stable and non-oscillatory solution for the convection-dominated diffusion problem. We develop patch-wise local projection stabilized conforming and nonconforming finite element methods for the convection-diffusion problems. The numerical experiments confirm efficiency of the proposed stabilization technique and validate the theoretical convergence rates.

Double diffusive mixed convection flow with exponentially decreasing main stream velocity

Nafisabanu Kumbarwadi
Karnatak University, Dharwad

In this talk, we present the double diffusive mixed convection flow over an exponentially decreasing main stream velocity in presence of chemical reaction and heat generation/absorption parameter. Non-similar transformations are used to reduce the boundary layer equations into dimensionless equations and further solved by implicit finite difference scheme in combination with quasi-linearization technique. The influence of various governing parameters: the heat generation/absorption parameter (Q), the ratio of buoyancy forces (N), the Richardson number (Ri), the chemical reaction parameter on the flow patterns are presented. Moreover, the effects of these parameters on skin-friction coefficient, Nusselt number and Sherwood number is also studied. The numerical investigation reveals that increase in heat generation/absorption parameter increases the thermal boundary layer thickness. Furthermore, an increase in the chemical reaction parameter reduces the thickness of the concentration boundary layer.

Numerical investigation of unsteady double diffusive MHD mixed convection from an exponentially stretching sheet

Shashikant
Karnatak University, Dharwad

This research work illuminates the influence of unsteadiness and the transverse applied magnetic field over the double diffusive mixed convection flow along an exponentially stretching sheet. The effects of non-uniform heat source/sink and nth order homogeneous chemical reaction between the fluid and the diffusing species are also examined. The non-dimensional equations in non-similar form are obtained from the non-linear partial differential equations which govern the flow, thermal, and concentration fields. The so obtained coupled non-dimensional equations are solved by the coalition of implicit finite difference scheme and the quasi-linearization technique. Furthermore, the skin-friction and heat transfer gradients are presented along with their graphical representations.

Coupled thermoelastic interactions with memory dependent derivatives under an exact heat conduction with a delay

Shashi Kant
IIT (BHU), Varanasi

A new model of coupled thermoelasticity based on memory dependent derivatives has been considered to investigate the problem of wave propagation in a homogeneous, isotropic and unbounded elastic solid. The bounding surface is taken to be stress free and subjected to a thermal shock. Laplace and Hankel transform techniques are used to obtain the solution of the different field variables like temperature, displacement, and stresses inside the medium. Detailed comparative analysis is also represented through the numerical results to see the effects of the kernels and time-delay parameters on the behavior of all the field variables.

Transient simulation of vapor-liquid two-phase flow inside single tube heat exchanger using finite difference method

Sandeep Malhotra
Institute of Technology, Nirma University

The vapor-liquid two-phase flow inside a heat exchanger is basically governed by the conservation laws of fluid dynamics which is a system of nonlinear partial differential equations. The model is capable to predict the transient behavior of the system and at the outcome we can analyze the behavior of temperature of the fluid inside a tube with the change in its phases. The study is important to predict the dynamic response of the system which is then useful to decide the proper feedback control system. To solve the system an efficient quasi-linearization approach is proposed with a suitable finite difference scheme and LU decomposition in this work. In the proposed work

Matlab is used to simulate the problem with the collaboration of Refprop software to call the properties of the fluid. In the present work, refrigerant R-22 is chosen as a working fluid to run the simulation. The work is capable to run the simulation for different refrigerants.

A new two-level implicit scheme based on cubic spline approximations for the system of 1D quasi-linear parabolic partial differential equations

Sachin Sharma
University of Delhi

In this work, we proposed a new two-level implicit method based on cubic spline approximations for the solution of the system of 1D quasi-linear parabolic partial differential equations subject to appropriately prescribed initial and natural boundary conditions. The proposed cubic spline method is derived directly from the continuity condition of the first order derivative of the cubic spline function. The stability analysis for a model problem is discussed. The method is directly applicable to problems in polar systems. The proposed method is tested to solve coupled Burgers' equations and Burgers-Huxley equation to demonstrate the strength and utility of the method. We show that the proposed method enables us to obtain high accurate solution for high Reynolds number.

Third order nonuniform mesh compact finite difference method adopted for two-dimensional elliptic equations

Neelesh Kumar
South Asian University

We developed a new third order nonuniform mesh compact finite difference method for the solutions of the two dimensional elliptic boundary value problems. This method is based on a geometric mesh where we have used nine-point spatial finite differences. The convergence analysis of the method has been achieved using the concept of matrix theory as well as graph theory. This scheme has been applied to a convection-diffusion equation and a semi-linear elliptic equation. The results confirm the third order accuracy of the method.

A new high accuracy half step cubic spline method for 1-D quasi-linear hyperbolic equations

Gunjan Khurana
South Asian University

In this work, we propose a new three level implicit numerical method based on half step cubic spline method of order two in time and four in space for the solution of one-space dimensional quasilinear hyperbolic PDE of the form

$$u_{tt} = A(x,t,u)u_{xx} + f(x,t,u,u_x,u_t).$$

We describe the half step cubic spline approximations and its properties. The new method is directly obtained from the consistency condition. The proposed method when applied to damped wave equation is shown to be unconditionally stable. Several benchmark problems have been solved and numerical results are provided to demonstrate the validity and utility of the method.

Unconditionally stable positivity preserving upwind scheme for multi-species transport with first order reaction network

L. Jones Tarcius Doss
Anna University

A finite volume formulation with a non-local approximation is applied to the system of advection-diffusion-reaction equations. A positivity preserving upwind scheme is derived as a result. Unconditional stability of the proposed scheme is proved. Further, the consistency is discussed analytically and illustrated numerically for different cases based on spatial and time step lengths. Numerical results show that the proposed scheme has higher order convergence when mesh ratio tends to zero. Numerical simulation for three dimensional contamination transport with advection flow in all three directions is also carried out with different types of sources.

Some exact solutions of breaking soliton system

Digvijay Tanwar
Motilal Nehru National Institute of Technology, Allahabad

Some new exact solutions of (2+1)-dimensional breaking soliton system of nonlinear partial differential equations are obtained by similarity transformations via Lie group theory. The infinitesimals are obtained using similarity transformation method to keep the breaking soliton system invariant and the system is reduced to a new system of partial differential equations with lesser number of independent variables, thereafter using similarity transformation method again the system of partial differential equations is reduced to a system of ordinary differential equations. Some explicit solutions of this system of ordinary differential equations are obtained under a suitable choice of function and making use of arbitrary constant. The graphical representation and physical analysis of solutions based on numerical simulations are discussed in the analysis and discussion section.

MHD flows of two-immiscible fluids through the channels filled with highly porous medium

Sneha Jaiswal
MNNIT, Allahabad

This talk is concerned with the flow of viscous, steady, incompressible, and immiscible fluids with different viscosities in the channels of two infinite parallel plates. The flow is driven by the

constant pressure gradient in the presence of transverse magnetic field of uniform strength. Both the channels are filled with the highly porous media and having different permeability. The flows through the channels are governed by the Brinkman equation with the inclusion of inertia term. No-slip conditions at the end of plates, continuity of velocity and continuity of shearing stress at the interface have been used as the boundary conditions to get the solution of the considered problem. The effect of various fluid parameters like permeability of porous region, magnetic number on the flow velocity profile, flow rate, and shearing stress has been discussed graphically.

A numerical solution for a Stefan problem with variable latent heat

Abhishek Kumar Singh

IIT(BHU), Varanasi

In this work, a one-phase Stefan problem with latent heat depending linearly on space variable is investigated. The Stefan problem involves a nonlinear boundary condition of the second type. The approximate analytical solution of the problem is obtained by Chebyshev spectral method based on operational matrix of derivatives. The scheme is applied to the problem which has a known similarity solution. The results obtained have been compared with existing exact solution in the literature and are in good agreement. The problem has been discussed in detail by considering different values of the parameters appearing in the problem. The temperature distribution and position of moving interface for different values of the parameters are shown by tables and figures. The results reveal that the proposed method is very effective and simple to solve the moving boundary problems.

Propagation of Rayleigh-type wave in an initially stressed heterogeneous transversely isotropic dissipative media

Shalini Saha

Indian Institute of Technology (Indian School of Mines), Dhanbad

The propagation of Rayleigh-type wave in an initially stressed heterogeneous transversely isotropic media has been delved in the present study. The expression for frequency equation has been obtained in closed form subjected to certain boundary condition. Numerical calculation and graphical illustration has been performed to study the effect of initial stress parameter, heterogeneity parameter, and viscoelastic parameter. It has been observed that the parameters have significant effect on the propagation of Rayleigh-type wave.

Influence of corrugated interface and poroelasticity on Rayleigh-type wave propagation

Anusree Ray
Indian Institute of Technology (Indian School of Mines), Dhanbad

The present problem deals with the propagation behaviour of Rayleigh-type wave through a fluid stratum mounted over a fluid saturated poroelastic corrugated substrate. The expression of dispersion relation is obtained analytically. The profound effects of corrugation and porosity are illustrated by means of graphs. Expression of dispersion relation when the common interface is devoid of corrugation is deduced in closed form as a special case of the problem. A comparative analysis is carried out to forefront the effect of presence of poroelasticity in the present study.

Propagation of Love-type wave in an imperfectly bonded piezoelectric layer with irregularity

Mriganka Shekhar Chaki
Indian Institute of Technology (Indian School of Mines), Dhanbad

The present paper investigates the propagation of Love-type wave in a piezoelectric layer imperfectly bonded with lower fiber-reinforced half-space with rectangular shaped irregularity at the common interface. With the aid of analytical methods along with perturbation technique, the closed-form expression of phase velocity of Love-type wave has been deduced for both electrically open and short conditions. Some special cases of the problem have also been studied and the obtained results are found to be in well-agreement with the classical Love wave equation. Numerical computations and graphical illustrations have been carried out to demonstrate the significant effects of various parameters viz. irregularity parameter, imperfectness parameter associated with common interface, dielectric constant, and piezoelectric coefficient on phase velocity of Love-type wave. Moreover, comparative study has been performed to unravel the effects of the presence of reinforcement, piezoelectricity, irregularity, and imperfectness in the composite structure which is a salient feature of the present study.

C^0 interior penalty method for a fourth order Dirichlet boundary control problem

Sudipto Chowdhury
Indian Institute of Technology Bombay

In this talk we consider a fourth order energy space based linear quadratic Dirichlet boundary control problem governed by biharmonic equation with clamped plate boundary condition. We show that the minimum energy in the minimization problem can be achieved with an equivalent $H^{3/2}$-norm of the optimal control on the boundary. We briefly discuss the C^0 interior penalty method based *a priori* energy norm and L^2-norm estimates for the control, state, and adjoint state

variables. If time permits I will discuss a few numerical experiments which ascertain our theoretical findings.

An orthogonal spline collocation Crank-Nicolson method for 1D parabolic singularly perturbed reaction-diffusion problems

Pankaj Mishra
South Asian University

In this work, quasi-optimal error estimates are derived for the continuous-time orthogonal collocation method and a discrete-time collocation method (the Crank-Nicolson method) for approximating the solution of a class of 1D parabolic singularly perturbed reaction-diffusion problems. The fitted mesh (Shishkin mesh) technique is employed to generate piecewise uniform mesh in spatial direction. The orthogonal spline collocation with C^1 splines of degree $r \geqslant 3$ is used on Shishkin mesh. The results of numerical experiments are presented to validate the theoretical analysis.